Exercise session: week 6

November 19, 2024

Let R denote a commutative ring; k a field and A a k-algebra. Exercises marked with $\star$ can be handed in for grading.

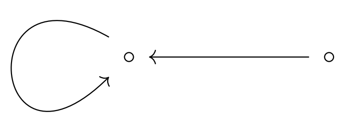
Exercise 1. For each of the following quivers, give a basis of the path algebra, then write the multiplication table of this basis and finally write de path algebra as a triangular matrix algebra: 1. $\circ \longleftarrow \circ \longrightarrow \circ$; 2. $\circ \longrightarrow \circ \longleftarrow \circ$; 3. $\circ \leftleftarrows \circ \longleftarrow \circ$. More examples can be found in the book. Is it always possible to write a path algebra as an upper triangular matrix algebra?

$\star$ **Exercise 2.** A ring is called hereditary if its minimal projective resolutions have length at most 1.

1. Show that the algebra $\Bbbk[t]/(t^3)$ is not hereditary.

2. Let Q be a finite quiver with no cycles. Show that $\Bbbk Q$ is hereditary.

$\star$ **Exercise 3.** Let $A = \begin{pmatrix} \Bbbk[t] & 0 \\ \Bbbk[t] & \Bbbk \end{pmatrix}$ and view A as a $\Bbbk$-algebra with the usual matrix multiplication. Show that $A \cong \Bbbk Q$ where A is the quiver



Exercise 4. Let P be a finite set equipped with a partial order relation $\leq$. Let x, y be elements of P. We say that $x \leq y$ is a *covering relation* if the interval $[x, y]$ of the poset contains exactly two elements. Define the *Hasse diagram* of the poset $(P, \leq)$ to be the directed graph with vertices the elements of P and arrows from x to y if and only if the relation $x \leq y$ holds and is a covering relation.

1. Consider the set $\{1, 2, 3, 4, 6, 12\}$ orderd by the relation $x \leq y$ if and only if x divides y. Draw its Hasse diagram. Compute its incidence algebra.

2. Show that the quiver algebra $\Bbbk H$ surjects onto the incidence algebra of the poset $(P, \leq)$. Give an explicit morphism of algebras.

3. Give a set of generators for the kernel of this map using paths in the Hasse diagram.